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On dielectric constant and loss tangent of dielectric material for circuit boards

— Difference in measurement methods and definitions and attentions need to be paid in applying them to circuit design —

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August 4, 2021

1 Introduction

Current high-speed communication systems, whether wireless or wired, demand every electric components to realize high performances in wider bandwidth at higher higher frequency band in order to achieve still higher communication capacity and quality. Accurate dielectric constant (Dk) and loss tangent (Df) from DC to millimeter wave band over 110 GHz now become the critical information for circuit designers. In addition, by improving the estimation accuracy of device characteristics at the design stage, the development cycle can be shortened, which is effective in reducing costs. Therefore, the exact Dk and Df values are also quite important information for the designers in this regard as well.

Under these circumstances, almost all companies supplying laminate materials used in printed circuit boards (PCBs) and integrated circuit packages provide their Dks and Dfs in various frequency bands. However, Dks and Dfs, and their effective values of circuits, are extracted according to different measurement methods, and they have different definitions. This situation sometimes can cause problems for circuit designers. For example, they are easily getting in lost in choosing which Dk and Df is adequate for the accurate simulation that well reproducing the actual device performances at each design stage, such as, transmission line design, selection of electronic materials, design of devices and modules, etc.

To reduce these issues, in this white paper we will clarify the measurement methods and the definitions for the Dk and Df over 10 GHz that we are providing, and will give precautions on their usages. In particular, we will quantitatively discuss the definition of effective dielectric constant, the difference in the effective dielectric constants arisen from the difference in its definition, and errors due to the measurement methods.

2 Dk and Df measured with a balanced-type circular disk resonator method[1]

Dks and Dfs (hereinafter, represented as $\varepsilon_r^{\text{BCDR}}$ and $\tan\delta^{\text{BCDR}}$, respectively) of laminate materials at frequencies more the 10 GHz, published in our product catalogue or on our home page, are measured with the Balanced-type Circular Disk Resonator (BCDR) method. The method is classified as one of the resonator methods, and it can accurately measure $\varepsilon_r^{\text{BCDR}}$ and $\tan\delta^{\text{BCDR}}$ from resonant frequencies and unload Qs for each resonant mode generated on the BCDR. Published $\varepsilon_r^{\text{BCDR}}$ and $\tan\delta^{\text{BCDR}}$ are of at room temperature, however, those in the temperature range from $-40\,°\mathrm{C}$ to $+150\,°\mathrm{C}$ can be also measured.

Electric fields in the BCDR are polarized perpendicular to the laminate sample under test, and uniformly spreads within the resonator [2], therefore, $\varepsilon_r^{\mathrm{BCDR}}$ and $\tan\delta^{\mathrm{BCDR}}$ come to show closer to the volume-averaged value of the dielectric constants and the loss tangents of the laminate components (namely, resin, filler and glass fabrics), respectively, for the perpendicularly-polarizing electric fields. Conversely, as it is clear from the polarization direction of the fields in the BCDR, the method is essentially unable to measure the dielectric constant and the loss tangent in the direction parallel to the laminate surface. In addition, as the BCDR must have the total of four copper surfaces (two GND surfaces and two [front and back] of the internally mounted disk-type electrode), then conductivity and surface roughness of the copper influence the extraction accuracy of $\tan\delta^{\mathrm{BCDR}}$.

As understood from the measurement principle, only if the PCB under design is composed of the transmission line that supports propagation modes with longitudinal polarization and a uniform field distribution in the perpendicular direction to the surface (ex., a circuit with substrate integrated waveguides), $\varepsilon_r^{\mathrm{BCDR}}$ and $\tan\delta^{\mathrm{BCDR}}$ can be applied as the physical characteristic valued of the laminate, and simulations executed at the design surely result in the good agreement with measurements. Conversely speaking, if the propagation modes in the circuit has a non-uniform electric field distribution in the thickness direction and have transversal polarization (such as a circuit consisting of microstrip lines patterned on a laminate with glass fabric), we recommend the designer to consider $\varepsilon_r^{\mathrm{BCDR}}$ and $\tan\delta^{\mathrm{BCDR}}$ as reference values, and to extract those from transmission properties of the line composing the circuit, however, it takes a time.

3 Dk and Df extracted from the microstrip line

For designers developing planar circuits consisting of quasi-TEM transmission lines such as microstrip lines (MSLs) and strip lines, we can also supplementary provide dielectric constant, $\varepsilon_r^{\mathrm{MSL}}$, and loss tangent, $\tan\delta^{\mathrm{MSL}}$, extracted from the MSL with a characteristic impedance of 50 Ω (line width is optimized at 79 GHz) constructed on the laminate under test (usually, dielectric thickness and copper foil thickness are fixed at 4 mil and 18 μm, respectively), in addition to the above-mentioned $\varepsilon_r^{\mathrm{BCDR}}$ and $\tan\delta^{\mathrm{BCDR}}$. The purpose of publishing $\varepsilon_r^{\mathrm{MSL}}$ and $\tan\delta^{\mathrm{MSL}}$ is reducing the work of the designers to extract them. The reason for setting the above extraction conditions is to measure the transmission characteristics under conditions close to actual use, and to estimate $\varepsilon_r^{\mathrm{MSL}}$ and $\tan\delta^{\mathrm{MSL}}$ by using the computational environment that will be applied to the design.

First, when measuring the transmission characteristics of the MSL, de-embedding is performed with using a multiline TRL calibration method by NIST [3] to extract accurately the propagation properties of the MSL section only. After acquiring S parameters from 1 to 110 GHz for the calibration standards (multiple MSLs having different line length, short and open standards) by using a prober system, we calculate a complex propagation constant γ from the phase of de-embedded S_{21} to find attenuation constant, α, and wave number, k^{eff}, corresponding to the real and imaginary parts of γ, respectively. Then, performing electromagnetic field analysis (finite element method is applied for this purpose) to the cross-sectional shape of the measured MSL as a simulation model, we consider the dielectric constant and the loss tangent that well reproduce measured k^{eff} and α at the extracted frequency as $\varepsilon_r^{\mathrm{MSL}}$ and $\tan\delta^{\mathrm{MSL}}$, respectively. Since this method is one of the transmission line method that does not use a resonator, it has an advantage that the test frequency can be set freely, although the accuracy of measurement is inferior to that of the BCDR method. In addition, the measured temperature defined by the capacity of the prober can be set in the range of $-40\,°\mathrm{C}$ to $+200\,°\mathrm{C}$. Note that with this method, the loss due to the roughness of the copper foil also affects $\tan\delta^{\mathrm{MSL}}$.

As described in the previous white paper "Why are two set of Dk and Df need for?," the effective dielectric constant measured directly from the MSL is generally the one not for the pure propagation mode of the MSL (namely, quasi-TEM mode), but for a coupling mode between the quasi-TEM mode and the lowest surface wave mode that can propagate on a dielectric substrate without metal strips

[4]. However, when identifying Dks and Dfs by the simulation, it is usually difficult to take the effects of the coupling mode into calculation [1] . If the electromagnetic mode propagating on the transmission line is the same between the actual line and the simulation model, then the extracted $\varepsilon_r^{\text{MSL}}$ are exactly the same as the dielectric constant for the laminate. However, this assumption does not hold true in our case, therefore, extracted $\varepsilon_r^{\text{MSL}}$ contains the effect of the coupling mode and is considered as an effective value. According to [4], the thicker the substrate is, and the higher the frequency is, the more remarkable the effect becomes. The difference between $\varepsilon_r^{\text{MSL}}$ and $\varepsilon_r^{\text{BCDR}}$ becomes remarkable in the frequency rage of approximately 70 GHz band or higher. Thus the effect of the coupling mode is contained in $\varepsilon_r^{\text{MSL}}$, however, not in $\varepsilon_r^{\text{BCDR}}$. Therefore, if the circuit design is performed with a circuit simulator that cannot treat the coupling mode, or that is not a full-wave analysis, using $\varepsilon_r^{\text{MSL}}$ rather than $\varepsilon_r^{\text{BCDR}}$ can improve the design accuracy, because $\varepsilon_r^{\text{MSL}}$ already contain the effect by the mode. Finally note that there are some circuit simulators that adopt empirical formulae that contain the effect by the coupling mode (the empirical formulae derived from the results by an full-wave analyses such as a spectral domain method [5]). When using the simulators, using $\varepsilon_r^{\text{BCDR}}$ rather than $\varepsilon_r^{\text{MSL}}$ may give results more consistent with reality.

4 Effective dielectric constant and effectiveness loss tangent of a transmission line

4.1 What is the effective dielectric constant?

Do you know that there are two definitions for the “effective dielectric constant”? To explain this, a uniform straight transmission line with the length of l is assumed, and It is assumed that the characteristic impedance of the line exactly matches the impedance of the two ports connected to both ends of the line, namely that impedance matching is completely realized. Under these assumptions, the S-parameters of this line can be expressed as $S_{11} = S_{22} = 0$ and $S_{21} = S_{12}^* = e^{i\gamma l}$. Here, γ is called the complex propagation constant and can be divided into wave number k of the real part and attenuation constant α of the imaginary part as $\gamma \equiv k + i\alpha$. When the wavelength of a sine wave with the frequency of f that propagates on this transmission line is assumed to be λ_g, the wave number k is defined as $k \equiv 2\pi/\lambda_g$, and the attenuation constant α is an amplitude attenuation, namely, a constant representing energy dissipation.

Then, if we focus only on the propagation property, namely, $e^{i\gamma l}$, and ignore the detailed structure of the line once, the electromagnetic wave propagating on the line can be considered to have effective complex permittivity, ${}^*\varepsilon_r^{\text{eff}} = \varepsilon_r'^{\text{eff}} + i\varepsilon_r''^{\text{eff}}$, or effective complex refractive index, ${}^*n^{\text{eff}}$, and to be equivalent to a planar wave, $e^{i\gamma l}$, propagating through an isotropic dielectric body with ${}^*\varepsilon_r^{\text{eff}}$. With this modeling, a dispersion relation, $2\pi f = \omega = ck_0 = (c/{}^*n^{\text{eff}})\ \gamma$ (c is the speed of light and k_0 is wave number in vacuum) is obtained. In addition, since a relation ${}^*\varepsilon_r^{\text{eff}} = ({}^*n^{\text{eff}})^2$ is established, a relational expression $(\gamma/k_0)^2 = {}^*\varepsilon_r^{\text{eff}}$ is obtained. From these relational expressions, it can be understood that effective dielectric constant $\varepsilon_r^{\text{eff}} \equiv \varepsilon_r'^{\text{eff}}$ and effective loss tangent $\tan\delta^{\text{eff}} \equiv \varepsilon_r''^{\text{eff}}/\varepsilon_r'^{\text{eff}}$ are given in the following equations.

$$\varepsilon_r^{\text{eff}} = \frac{\text{Re}\ \gamma^2}{{k_0}^2} \tag{1}$$

$$\tan\delta^{\text{eff}} = \frac{\text{Im}\ \gamma^2}{\text{Re}\ \gamma^2} \tag{2}$$

[1]When analyzing straight and uniform MSLs by electromagnetic simulation, it is common to create a simulation model surrounded by a parallelepiped boundary with two ports and four perfect conductor walls to reduce computational costs. If the boundary is set so as to be a cut-off waveguide in the simulation frequency range, there exists neither surface wave modes nor the coupling mode. Therefore, $\varepsilon_r^{\text{MSL}}$ and $\tan\delta^{\text{MSL}}$ are optimized for quasi-TEM mode so as to reproduce measured k^{eff} and α, but these are basically those of coupling mode.

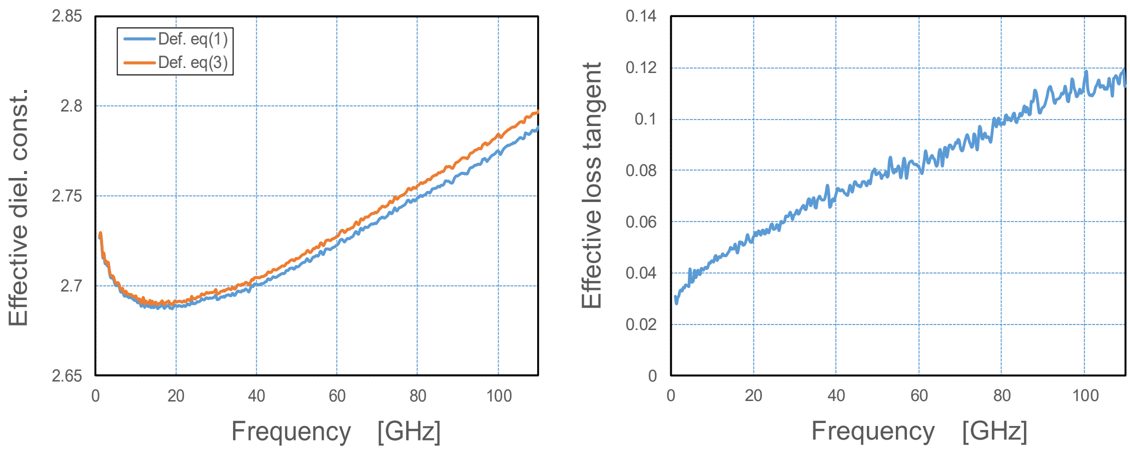

Figure 1: (Left) Frequency dependencies of the effective dielectric constants according to Definitions (1) and (3) (denoted as "Def. eq(1)" and "Def. eq(3)", respectively) calculated from S_{21} for a MSL after de-embedding. (Right) Frequency dependencies of the effective loss tangent calculated by Equation (2). It can be seen in the left figure that the values according to Definition (3) always show larger values than those according to Definition (1). In order for the difference in effective dielectric constants between the two definitions to be remarkable, the MSL must have a large effective loss tangent shown in figure (Right).

As can be seen in Equation (1), α in addition to k is incorporated into the effective dielectric constant, $\varepsilon_r^{\text{eff}}$, according to this definition, since $\varepsilon_r^{\text{eff}} \propto \text{Re } \gamma^2 = k^2 - \alpha^2$.

As can be seen from the above derivation, Equation (1) is physically straightforward expression as the definition of the effective dielectric constant, but the following definition which excludes α from Equation (1) is commonly used, in order to simplify the discussion.

$$\varepsilon_r^{\text{eff}} = \frac{(\text{Re } \gamma)^2}{{k_0}^2} \tag{3}$$

In the following, we adopt Equation (3) as the definition of the effective dielectric constant. For a low loss transmission line, that is, α is sufficiently smaller than k, Equation (3) approximately coincides with Equation (1). However, when α is sufficiently large compared with k, differences between the two definitions become remarkable. The higher the frequency becomes, the more the effective dielectric constant (1) tends to decrease compared with (3), because α for a common transmission line shows the bigger values as the frequency become higher, and because of $\varepsilon_r^{\text{eff}} \propto \text{Re } \gamma^2 = k^2 - \alpha^2$. These qualitative behaviors of the two definitions can be seen concretely from Figure 1. Figure 1 (Left) shows the frequency dependencies of the effective dielectric constant in accordance with two definitions, and theses are extracted from de-embedded S_{21} for a MSL, which patterned on the 4 mil thick substrate and has large transmission losses. As can be seen from the figure, the values extracted by Definition (1) are smaller than the values by Definition (3) at all frequencies. However, it should note that the differences between the two effective dielectric constants become remarkable as shown in Figure 1 (Left), only when the effective loss tangent defined in Equation (2) becomes large as shown in Figure 1 (Right). Also, it can be seen in the figure that the difference between the two effective dielectric constant becomes more remarkable at higher frequencies. This is because α commonly show larger value at frequencies higher than millimeter wave band. Therefore, designers need to pay attention to which definition is used to determine the effective dielectric constant especially in such frequency range. More specifically, when determining the dielectric constant of a laminate under test from the effective one measured by using the method described in Section 3, it is expected that extracted dielectric constant shows greater than the actual value, if Definition (3) is used for calculating the effective one from the de-embedded S_{21}.

4.2 Effective dielectric constant of the microstrip line

The effective dielectric constant $\varepsilon_r^{\text{eff}}$ for the microstrip line is commonly calculated using Equation (3). As assumed as a prerequisite for the calculations in subsection 4.1, calculating the effective dielectric constant according to Equation (3) is possible only if " the impedance matching is completely realized." Therefore, the de-embedding method described in Section 3 is applied to satisfy this prerequisite. As denoted in Subsection 4.1, $\varepsilon_r^{\text{eff}}$ includes the propagation properties of the microstrip line in addition to the properties of the materials constituting the laminate under test (resin, copper foil and fillers); therefore, It cannot be used directly as a physical property of the laminate input to the circuit/electromagnetic field simulator applied to the design. However, it can be used for an ideal transmission line model directly parameterized by the effective dielectric constant if the designer can be used a circuit simulator that supports the model. Then designers can perform more realistic and accurate optimizations by inserting $\varepsilon_r^{\text{eff}}$ and α into the model.

5 On the measurement of the effective dielectric constant using the microstrip differential phase length method [6]

As the final topic, we would like to describe a method for measuring the effective dielectric constant, "microstrip differential phase length method," which is different from the one described in Section 4. The measuring steps for this method are as follows: Firstly measuring two phases (which is define as the argument of S_{21}), φ_0 and φ_1, for two microstrip lines that have the identical line structure but have different line lengths (difference in line length: Δl). Next calculating the phase shift per unit length $\theta = \Delta\varphi/\Delta l$ from the phase difference in $\Delta\varphi = \varphi_1 - \varphi_0$. At this step, $\varepsilon_r^{\text{eff}}$ can be obtained from Equation (3) since $\theta = \text{Re}\ \gamma$.

Emphasizing here that the impedance matching is completely realized," which is the repeat of what was said in Section 4, so as that the relation $\theta = \text{Re}\ \gamma$ holds true. However, in actual measurement, measurement is usually performed under conditions such that this precondition is not sufficiently satisfied. We discuss below how so-called impedance mismatch affects the calculation of the effective dielectric constant by Equation (3). For this aim, as similar to the previous section, we assume an ideal transmission line with a length l and a propagation constant $\bar{\gamma}\,(\equiv -i\gamma)$.

To identify the phase part of S_{21} when impedance mismatch, S_{21} under $Z_0 \neq Z$ is given as (4) in accordance with a transmission line theory [7]. In the equation, Z is the characteristic impedance of the line and Z_0 is the impedance of the port.

$$S_{21} = \frac{2ZZ_0}{(Z^2 + {Z_0}^2)\sinh(\bar{\gamma}l) + 2ZZ_0\cosh(\bar{\gamma}l)} \tag{4}$$

In the following, to consider a situation where the impedances match is slightly broken, that is, $Z = (1 + \Delta)Z_0$ ($|\Delta| \ll 1$), is substituted into the above equation. Then, if we arrange Equation (4) with an approximate calculation at $|\Delta| \ll 1$, assuming a planar wave propagating in a medium with a frequency dispersion, namely, $S_{21} \propto \exp(-\bar{\gamma}^{\text{eff}} l) = \exp[\,(ik^{\text{eff}} - \alpha^{\text{eff}})\, l\,]$, we get Equation (5).

$$
\begin{aligned}
S_{21} &= \frac{e^{-\bar{\gamma} l}}{1 + \delta e^{-\bar{\gamma} l} \sinh(\bar{\gamma} l)}, \quad \text{where } \delta \equiv \frac{\Delta^2}{2(1+\Delta)} \\
&\approx [\,1 - \delta e^{-\bar{\gamma} l} \sinh(\bar{\gamma} l)\,]\; e^{-\bar{\gamma} l} \\
&= \left(1 - \frac{1}{2}\delta\right) \left(1 + \frac{\delta}{2-\delta}\; e^{-2\bar{\gamma} l}\right) e^{-\bar{\gamma} l} \\
&\approx \left(1 - \frac{1}{2}\delta\right) \left(1 + \frac{\delta}{2}\; e^{-2\bar{\gamma} l}\right) e^{-\bar{\gamma} l} \\
&\approx \left(1 - \frac{1}{2}\delta\right) \exp\left[\; (\delta/2)\; e^{-2\bar{\gamma} l}\; \right]\; e^{-\bar{\gamma} l} \\
&= \left(1 - \frac{1}{2}\delta\right) \exp\left[\; -\bar{\gamma} l + (\delta/2)\; e^{-2\bar{\gamma} l}\; \right]
\end{aligned} \tag{5}
$$

Then, the imaginary part is extracted from the phase part of Equation (5) and divided by the line length l to find k^{eff}, and we get Equation (6).

$$
\begin{aligned}
k^{\text{eff}} &= \frac{1}{l}\; \text{Im}\left[\; -\bar{\gamma} l + (\delta/2)\; e^{-2\bar{\gamma} l}\; \right] \\
&= k + \frac{\delta}{2l}\; e^{-2\alpha l} \sin(2kl)
\end{aligned} \tag{6}
$$

When the line propagation loss is small, in other words,$e^{-2\alpha l} \approx 1$, then $|\, e^{-2\alpha l} \sin(2kl)\,| \approx 1$ because of $|\sin(2kl)| \leq 1$. Thus, when it is measured under the condition where the impedance is unmatched, it is inferred that k^{eff} contains an error, approximately $\Delta k^{\text{eff}} = \delta/(2l) = \Delta^2/\,[\,4l\,(1+\Delta)\,] \approx \Delta^2/(4l)$, according to Equation (6). Note that since there is a relationship shown in Equation (3), the error of the effective dielectric constant $\Delta\varepsilon_r^{\text{eff}}$ is calculated as follows from the propagation relationship of the error.

$$
\begin{aligned}
\Delta\varepsilon_r^{\text{eff}} &= \left| \frac{\partial \varepsilon_r^{\text{eff}}}{\partial k} \right| \Delta k^{\text{eff}} \\
&= \left| \frac{\partial}{\partial k} \left(\frac{k}{k_0}\right)^2 \right| \Delta k^{\text{eff}} \\
&= 2\; \frac{\Delta k^{\text{eff}}}{k_0} \sqrt{\varepsilon_r^{\text{eff}}} \\
&= \frac{c\,\Delta^2 \sqrt{\varepsilon_r^{\text{eff}}}}{4\pi f l}
\end{aligned} \tag{7}
$$

With Equation (7), the measurement error of the effective dielectric constant resulting from impedance mismatch can be calculated. We assumed using our prober system and measured the MSL with $l =$ 20 mm and $\varepsilon_r^{\text{eff}} \approx 3$ under the impedance mismatching condition where $|S_{11}|$ was approximately $-$ 10 dB (since $|S_{11}| \approx |\,(Z_0 - Z)/(Z_0 + Z)\,| \approx \Delta/\,2$, then $\Delta \approx 0.63$). We were able to obtain the maximum frequency, f, where a 10 % or larger measurement error of the effective dielectric constant ($\Delta\varepsilon_r^{\text{eff}}/\varepsilon_r^{\text{eff}} \geq 0.1$) occurs, and f was approximately 2.7 GHz from Equation (7). As seen with this result, it is found that the effect of the impedance mismatch appears in the measurement accuracy of the effective dielectric constant in the frequency band of 10 GHz and under. Therefore, when the effective dielectric constant at or under 10 GHz is measured using this method, it is expected to take the measurement using a line as long as possible after having achieved impedance matching sufficiently to improve the accuracy of the measurement.

To verify this result, we created an impedance mismatch condition with $|S_{11}| = -10$ [dB] by changing the impedance of the measurement port from 50 Ω using S parameter (S parameter under the impedance matching condition) after de-embedding measured from the MSL with 20 mm-length to calculate the effective dielectric constant according to Definition (3). The results are shown in Figure 2. As can be seen from the figure on the left, measurement errors that behave like a damped oscillation occur in the effective dielectric constant under the impedance mismatch condition. This is the effect of damping oscillation factor $e^{-2\alpha l}\sin(2kl)$ [second term on the right side of Equation (6)]. As mentioned above, the frequency where a 10 % error occurred was 2.7 GHz based on the calculation using Equation (7). However, as can be seen in the figure on the right, it was in the neighborhood of 1.8 GHz in this calculation example where $\Delta\varepsilon_r^{\mathrm{eff}}/\varepsilon_r^{\mathrm{eff}} \approx 0.1$, showing good agreement in the range of an approximation applied at the time of derivation. Based on the results thus far, the discussion in this section was confirmed with a numerical experiment using the actual measurement results.

In summary, it is essential "to completely match the impedance between two ports connected to the transmission line and its ends" to improve the accuracy of the measurement of the effective dielectric constant. When this condition is not ensured, in other words, when it is in an impedance mismatch state, the measurement error occurs in the low frequency band, but can be reduced by using a long line.

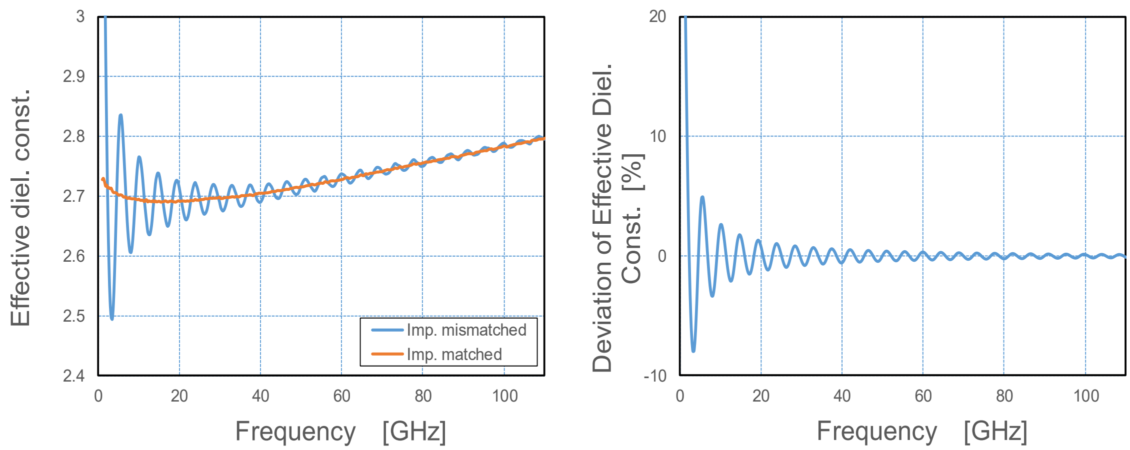

Figure 2: (Left) [Imp. matched] Frequency characteristics of the effective dielectric constant according to Definition (3) derived from 20 mm-long MSL after de-embedding. [Imp. mismatched] Frequency characteristics of the effective dielectric constant. Calculated from S parameter after the port impedance was converted ($Z_0/Z \approx 0.52$) so that S parameter after de-embedding became $|S_{11}| = -10$ [dB]. (Right) Frequency characteristic of $\Delta\varepsilon_r^{\mathrm{eff}}/\varepsilon_r^{\mathrm{eff}}$. As can be seen in the figure on the left, measurement errors that behave like a damped oscillation occur in the effective dielectric constant, when there is an impedance mismatch. In addition, as can be seen in the figure on the right, the errors become remarkable in lower frequency band. Note that, although the frequency where an error of 10 % occurred was 2.7 GHz based on the calculation using Equation (7), it was in the neighborhood of 1.8 GHz in this calculation example where $\Delta\varepsilon_r^{\mathrm{eff}}/\varepsilon_r^{\mathrm{eff}} \approx 0.1$.

6 Summary

In this technical report, we explained determination methods for the dielectric constant and loss tangent in each frequency band that we provided and points of attention derived from the determination methods that become necessary when those methods are used in circuit design. We have published the dielectric constant and loss tangent that were determined mainly by a balanced-type circular disk resonator method, but we can also supplement the dielectric constant and the loss tangent derived from a microstrip line. In addition, the device is also capable of evaluating temperature dependency of those values. Note that we also explained the necessity of determining the value to be applied as a substrate property value after having considered the distribution of the electromagnetic field in a line propagation mode when a distribution constant circuit is designed under a circuit / electromagnetic field analysis environment. In addition, we mentioned that there were two definitions for the effective dielectric constant and clarified that the difference between the two became remarkable in high frequency millimeter wave bands or above in a line with large loss. Furthermore, we explained that impedance matching between the port and line is important to improve the accuracy of the measurement of the effective dielectric constant, and quantitatively discussed that the measurement error becomes remarkable at the low frequency band when this condition could not be ensured.

References

[1] H. Kawabata, and Y. Kobayashi, "The analysis of a balanced-type circular disk resonator excited by coaxial cable lines to measure the complex permittivity," APMC 2001. 2001 Asia-Pacific Microwave Conference (Cat. No.01TH8577), DOI: 10.1109/APMC.2001.985379, Dec. 2001.

[2] J. Nakatsutsumi, Y. Kobayashi, and Z. Ma, "Discussions on measurement accuracy of complex relative permittivity using a balanced-type circular disk resonator method," 2014 Asia-Pacific Microwave Conference, 2014, pp. 522-524.

[3] D.C. DeGroot, J.A. Jargon, and R.B. Marks, "Multiline TRL revealed," 60th ARFTG Conference Digest, DOI: 10.1109/ARFTGF.2002.1218696, Dec. 2002.

[4] H.A. Wheeler, "Transmission-Line Properties of Parallel Strips Separated by a Dielectric Sheet," IEEE Trans. Microw. Theory Tech., MTT-13, Issue: 2, pp. 172–185, Mar 1965, DOI: 10.1109/TMTT.1965.1125962.

[5] T. Itoh and R. Mittra, "Spectral-Domain Approach for Calculating the Dispersion Characteristics of Microstrip Lines (Short Papers)," in IEEE Transactions on Microwave Theory and Techniques, vol. 21, no. 7, pp. 496-499, Jul. 1973, doi: 10.1109/TMTT.1973.1128044.

[6] Nirod K. Das, Susanne M. Voda, and David M. Pozar, "Two methods for the measurement of substrate dielectric constant," IEEE Transactions on Microwave Theory and Techniques, Vol. MTT-35, No. 7, July 1987.

[7] P.A. Rizzi, *Microwave Engineering: Passive Circuits*, Prentice-Hall Of India Pvt. Limited, ISBN: 9788120314610, 1988.